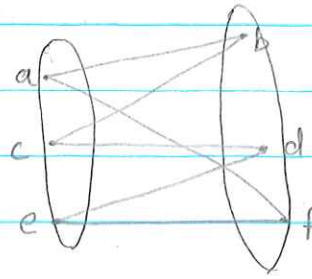
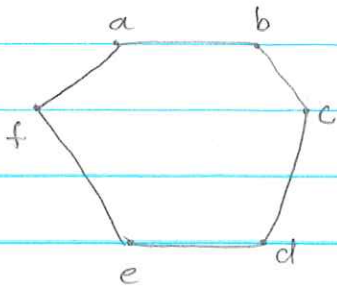


Solved Question Paper for block 1 unit 2.

(X) Q)

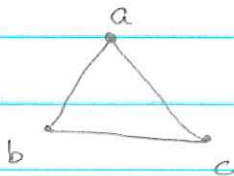
Show that C_6 is bipartite and K_3 is not bipartite.

Ans: C_6 .



From the above diagram it is clear that C_6 is bipartite.

K_3



K_3 is a complete graph, each vertex is connected to every other vertex. So, K_3 is not bipartite.

(X) Q)

show and explain that if G_1, G_2, \dots, G_n are bipartite graphs, then

$\bigcup_{i=1}^n G_i$ is bipartite.

Ans: Let $G_i, 1 \leq i \leq n$ be bipartite graphs with the bipartitions $V(G_i) = X_i \cup Y_i$, respectively. Let $G = \bigcup_{i=1}^n G_i$. Then $E(G)$ is the disjoint union $\bigcup_{i=1}^n E(G_i)$.

$V(G) = A \cup B$, where $A = \bigcup_{i=1}^n X_i$ and $B = \bigcup_{i=1}^n Y_i$, is a bipartition of $V(G)$. This can be seen as follows:

Let e be an edge in $E(G)$. Since $E(G)$ is a disjoint union of

①

$E(G_1), \dots, E(G_n)$, the edge e belongs to only one of them. Without loss of generality, suppose $e \in E(G_n)$. Since G_n is bipartite with a bipartition $X_n \cup Y_n$, this means e has one end vertex in X_n and the other in Y_n , that is e has one end vertex in A and the other in B .

Thus, G is bipartite with a bipartition $A \cup B$.

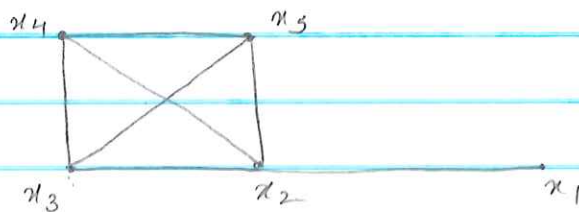
Q) Is a tree a bipartite graph? Justify.

Ans: Refer notes.

Q) Find the number of vertices and number of edges in a complete bipartite graph $K_{m,n}$.

Ans: Refer notes.

Q) Consider the graph on 5 vertices and 7 edges given in the figure. Find x_1 to x_5 walks of length 8 & length 4 respectively.



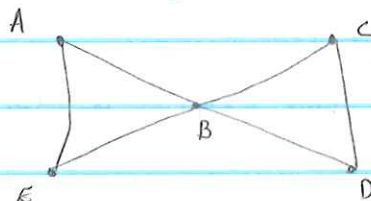
Ans: $l(w) = 8$.

$w = \{ x_1, x_1x_2, x_2, x_2x_3, x_3, x_3x_4, x_4, x_4x_2, x_2, x_2x_3, x_3, x_3x_4, x_4, x_4x_2, x_2, x_2x_5, x_5 \}$

$l(w) = 4$.

$w = \{ x_1, x_1x_2, x_2, x_2x_4, x_4, x_4x_2, x_2, x_2x_5, x_5 \}$

Q) Draw three spanning trees of the following graph:



Ans: refer notes.

Q) Define a bipartite graph. For which value of n is Q_n bipartite.

Ans: Definition of bipartite graph: refer notes.

Q_n is bipartite for any n . Let V_1 consists of all vertices whose sum of coordinates is odd and let V_2 consists of all vertices whose sum of coordinates is even. Two vertices in Q_n are connected if and only iff their coordinates differ in only one position, therefore the sum of their coordinates have different parity, so they are in different sets.

Q) Define tree and bipartite graphs. Is tree a bipartite graph?

Justify your answer.

Ans: Refer notes.

Q) Show that C_6 is a bipartite graph.

Ans: Refer first question of solved question paper.

Q) For which value of m and n is $K_{m,n}$ a tree?

Ans: Refer notes.

~~Q)~~ Draw a graph (connected) which can be both regular and bipartite?

Ans: logic is same as when $K_{m,n}$ is regular.

if $m = n$ the graph is regular.

refer notes.

Q) Which of the following graphs are trees, and why?

Ans: refer notes.

Q) Define walk, path and circuits in a graph.

Ans: refer notes.

~~Q)~~ Define a spanning tree: Illustrate with an example and a non-example each.

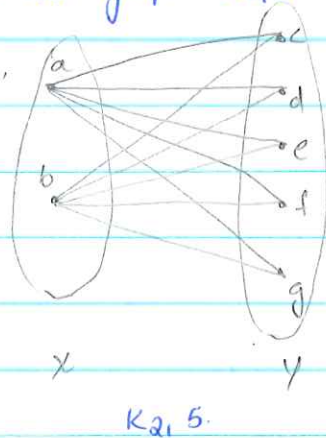
Ans: refer notes.

Q) Define bipartite graph. Also give an example of it.

Ans: Refer notes.

Q) Draw the graph $K_{2,5}$.

Ans:



Q) Define the concept of spanning tree for a graph G . Give a suitable example to illustrate the concept.

Ans: Refer notes.

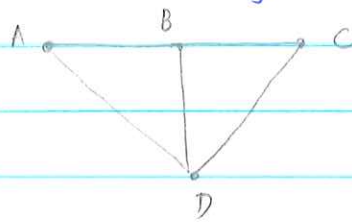
Q) Which connected graphs can be both regular and bipartite and why?

Ans: Refer notes.

Q) Define spanning tree with an example.

Ans: Refer notes.

Q) Draw three spanning trees of the graph.



Ans: Refer notes.